

Causerie on the Math GRE REU 2018

Jon Dewitt and Michael Neaton

August 29, 2018

1 Introduction

This note will cover some of our thoughts on the math subject GRE. This includes: (1) our thoughts on the test itself, (2) our philosophy behind the preparatory material we have created, (3) a brief summary and review of some other preparatory material, and (4) a brief note on how we believe one should study.

Instead of linking to the various sources to which we refer, all links may be found on our site for GRE prep, which is <https://math.uchicago.edu/~min/GRE>.

2 The Math GRE: Our Thoughts

The GRE subject test in mathematics is an exam intended for those who know the curriculum of a mathematics undergraduate student, and is used to determine eligibility in admission to graduate programs in mathematics, although mostly the United States. It is 66 questions long and lasts two hours and fifty minutes. It is offered three times a year, in September, October, and April.

The test is mostly computational calculus. Thus, one should make sure they know multivariable calculus in \mathbb{R}^3 . This constitutes about 50% of the exam. Another 25% of the exam comprises what is called “algebra”, which is split nearly evenly between linear algebra and “abstract” algebra (e.g., basic group/ring theory and basic number theory). The last 25% of the exam is a weird assortment of topics, including basic point-set topology, probability, statistics, geometry, and (real/complex) analysis.

One should take the exam in April of their junior year (e.g., April the year before they graduate). This gives sufficient time to get scores back and see if they are as high as is desired. Then it can be re-taken in September. If, instead, September is the first time the test is taken, then there is insufficient time to get scores back before October. Specifically, the deadline for signing up for the September test (for the minimum cost) is frequently before the September test date.

Most schools that accept the test are willing to wait to evaluate the application until after the October scores come in. There are a couple schools that claim this damages the application, and say they prefer the September test date; this claim sounds ridiculous, but it is even more reason to prefer the September test over the October one. Also note schools only look at the highest test score, as you choose which scores to report.

The raw test score is converted to a reported score range of 200 - 990. The higher scores are never given out; a better approximation of the top score is 920, but this is still too high on some exams. It doesn't matter too much, since this score is basically irrelevant; what matters is the percentile score.

We are frequently asked what score is good enough. This depends on many factors, including: your undergraduate institution, your grades, your coursework, extracurricular mathematics, your letters of recommendation, your statement of purpose and other essays, and what school you want to get into. As your undergraduate school becomes more prestigious (or, at least, has a standardized and meaningful grading policy), grades become more important and the test thus becomes less important. If you went to a liberal arts college, then the test is much more important, as it is the only in-class comparison that can be made about your abilities.

Thus, we cannot give general advice that is applicable to everyone about what a “good” score is. That said, the general understanding is that above 80th is “good”, above 90th is “great” and above 95th is “excellent”. Our impression is that most people ignore differences in score between students who rank above 90th percentile, since the test is not good at differentiating such students anyway.

Finally, despite how important the above rambling about the test may make it appear, it is our opinion that the test is actually a waste of time. The only reason to take the test is because it is required; the test is not actually meaningful in predicting how well one will do in graduate school. Most of the topics on the exam are irrelevant to most avenues of research in mathematics. (As a short aside, we make no claims of having a better system in mind at how to compare students of potentially disparate backgrounds. It is true that this test is the only way that students with unusual backgrounds in mathematics can be favorably compared to other students, short of having publications. We simply would have felt remiss if we did not point out that the test does waste time, with little added personal value.)

That being said, the topics on the exam are pretty much assumed knowledge in graduate school. And if there is anything that we’ve learned in our time in graduate school, it’s that life would be easier if we were better at calculus.

3 Our Material: Our Philosophy

We (with the help of a couple other graduate students) have created somewhere around 140 practice questions for the GRE. A brief comparison between released tests and our material shows that our material is far more difficult than the actual exam. Thus, some explanation of our approach is due.

First, there are many sources for actually learning the material on the exam. A majority of the exam is calculus, and there are literally thousands of textbooks covering calculus. None of the other topics are very specialized, so any introductory textbook would cover the material fine. Therefore, none of our preparatory material is meant to actually teach *mathematics*. It is an assumption that if there is a topic that one is not familiar with (e.g., differential equations), then one will go find a textbook on this topic and read until one knows how to solve the question.

This is the first purpose of our questions. If you do not know the theory behind one of our questions, then you may want to brush up on it. Working questions is a good way of determining which topics you do not need additional study in, and which topics you should learn better.

Second, our problems were created for the University of Chicago REU. Thus, there is a running assumption that the test-taker is a strong student. Therefore, rather than waste these students’ time by giving them simple questions, we have prepared more difficult questions based off the same principles. This has the added benefit of the questions not feeling recycled; though they may feel familiar, they tend to not yield the same approach. Examinees should get used to questions like this, so that they can be appropriately handled during the exam. Rather than feeling terrified that the solution isn’t immediate, this practices calming analyzing the question.

Third, our questions practice *skipping questions*. If, in your practice, you answer every question

in order, then you will attempt the same thing on the exam. But on the exam, you may find a question that you can't answer for some reason. Thus, you might waste ten minutes trying to answer it before deciding to move on. If you instead practice time management with difficult questions, you learn that skipping questions is a viable strategy.

Fourth, our questions are meant to practice *test-taking skills*. A lot of our questions are very difficult to answer directly, or to prove that the answer is actually correct. On a multiple-choice test, this is unnecessary. One need only eliminate four options to conclude the fifth. Thus, estimation of integrals is a viable strategy. In a similar vein, if there is a very difficult question, you still may be able to eliminate three of the options by some simple argument. Then, picking between the remaining two options is a 50/50 guess, which greatly improves your odds (and, thus, your score). These test-taking strategies are good to practice, and are much easier to practice with difficult questions.

Finally, some of our questions emphasize specific question types, tricks, or skills that we, or somebody we know, have seen on the actual exam. For example, the question "Determine the number of real zeros of f on the interval $[a, b]$," with f a polynomial and a, b real numbers, is a question that has actually appeared on the test. If your first time seeing such a question was on the exam, you may freeze; similarly, if the first example of such a question was really easy (e.g., with a quadratic), then you may have breezed past it in practice but may stumble on a more difficult question. Thus, we put such questions on for exposure, so you are familiar with certain types of questions.

Hence, we have pursued this approach as it fills a niche in the preparation materials. If you disagree with our philosophy or found our materials unhelpful, feel free to email me. Similarly, email me if you found the materials helpful, and would like to tell us why, if there are subjects or particular questions that we have not covered, or you have some comments on how to improve the material. If you are unable to solve a problem, or there is a typo, you can also email me at min@math.uchicago.edu.

4 Other Available Material: A Brief Review

Here, we review some of the other available materials out there. These reviews are brief, as we have not gone through all of the questions and notes provided. We can't waste all of our time on this, only an amount justifiable to our advisers. These will go in order as listed and linked on our GRE page.

- Charles Rambo has released a single test. It is available, with answers, for free online. He also has a solution manual. I can recommend this book, as I like the hippie theme.

I believe the test has been criticized for being too straight-forward, without enough tricks. I disagree with this analysis; I solved almost every single question via some trick, since I was too lazy to compute. The tricks are a little more subtle, while the questions are still possible to solve. If anything, this makes it quite similar to a majority of the actual GRE questions.

- The UCLA bootcamp is well-known. They have a giant compendium of information, currently spread across two different people's websites.

This is solid, well-organized material. I don't recall how much overlap there is with other material; some problems appear to be lifted from other sources. However, it is definitely a good source. I give it 4.5/5.

Do note that the provided “dubiously-relevant” exams are the same as the REA book. They are somewhat poorly copied, cutting off some of the question and some answer options. But this could give you an idea of what the REA book has before purchase.

- UCSB has a smaller program, similar to UCLA. I’ll add more here, if I look at it in detail.
- Gilad Pagi is seemingly attempting to create a market for this sort of thing. I cannot recommend that you do the online courses, purely because I myself am unwilling to purchase it to see how good it is.

The tests, you can purchase if you desire to. I have a copy of both tests. They are a bit lower quality than some available materials out there; specifically, the questions tend to be more straight-forward and a bit too focused than they should be. My guess is this is how Gilad studied and did well: he literally solved and computed everything, and learned much more material than necessary. This is fine, just substantially different from my own method; his tests would be a fine addition to your collection.

- The Princeton Review book is the quintessential *book* for studying for the Math GRE. Some people seem to think it is the only book, and it is definitely the most recent. If you want to buy anything that actually covers what is on the exam, my suggestion is this book.

The book comes with 2 tests, as well as some practice problems. The book is split into appropriate sections, and approximates what is on the exam fairly well. The last couple of sections become a bit more iffy, with the final section having quite a few typos; thus, you may need a different source for the more advanced topics. This is still the best book one can get.

- The REA book is the other book that people know about. It was published in 1989, so it is a bit too old. This means it is imbalanced (like how the older released GREs cover the wrong material; for example, there is far too much emphasis on approximation of functions and modeling data sets). It has 6 practice tests, and is still great practice for calculus (which is still 50% of the exam). The questions are a little odd, not quite similar to GRE style; I don’t know how to word what is off about them. If you are hunting for GRE-esque calculus questions, this book would be a fine addition.
- There is also Morris Bramson’s book, which not too many people seem to know about. This is slightly older, published in 1977. The same approximate statements as for the REA book apply to this book. It has 5 tests. The questions are actually quite good, so my advice about it is similar to that of the REA book. Note it contains fewer completely irrelevant questions (e.g., the REA’s emphasis on Simpson’s method), so is more highly focused to the practice of calculus.

5 How to Study: Our Opinion

Here, we give some idea of how to study. Note that what you should actually do depends on your background. Thus, we will be working from an assumption of an average somewhat advanced student in mathematics, e.g., you took a proof-based calculus course, but it was a while ago so you may not remember a lot of it, and also never really learned how to compute. After this, you took random courses that may or may not be relevant to the test.

1. Learn multivariable calculus in \mathbb{R}^3 , with a focus on computation.

2. Identify specific minor topics that you have not learned (e.g., differential equations, point-set topology). Pick up a basic source on this, and read the first couple chapters to get an idea of the subject. Make sure to solve some actual (computational, hopefully) exercises. One of the preparatory books could be helpful for this.
3. Do a couple of the practice tests, from one of the provided sources above (NOT one of the released tests).
4. Identify topics that you may need additional study in, and learn these.
5. Take the first two released tests (GR8767 and GR9367), with timing yourself. Identify which questions slowed you down, and which topics you still need to learn better.
6. Go and learn the topics you did not learn.
7. To get better at speed, do a ton of exercises in the topics you were too slow at. Like doing *all* of the exercises in some calculus textbook or something.
8. Go to our exercises, and do the first couple weeks. Time yourself, and try to finish quickly. Note how many questions you finished in an hour. Remember you should be guessing and skipping questions.
9. Figure out how to actually solve the questions you didn't know how to do. Figure out the tricks in the questions that either you took too long on, or did not know how to solve.
10. Fill in holes in knowledge as above.
11. Take the next released test, GR9768, under full test restrictions.
12. Again, fill in any holes that exist.
13. Rinse and repeat with alternating our questions and the released tests, each time learning the stuff you did not know.

As a note, if you did all of this studying for the test in April and now want to study more for the test in September, you may have an issue. Specifically, you have now seen all of the material before, and this may make it seem easier (e.g., you are now out of released tests). If you do not have a photographic memory, you can just wait a month or so and the questions should seem mostly new again. Thus, my suggestion would be to use the preparatory material that you have *not* yet used building up to the exam, and then to take GR1268 shortly before the exam. Hopefully it will seem mostly new, and will be good practice.

If you have a (near) eidetic memory and plan on taking the test twice, you may want to hold off on using GR1268 (or whatever the latest released test is) until before you take it the second time.

After you take the test in April, you should learn from it like any other practice test. Immediately after, take note of what you did not know and struggled with; try to recall questions you could not answer. After all, the actual exam is the best approximation of the exam.